CHAPTER III

EXISTING CONTINUOUS CARTOGRAM METHODS

3.1 Overview of Desired Characteristics

Several algorithms exist to construct continuous area cartograms, hereafter referred to as cartograms. Gusein-Zade and Tikunov, two researchers in the Department of Geography at Moscow State University, compiled a detailed review of current cartogram methods [10]. Their review introduces several desired properties that, if realized, would eliminate some of the shortcomings experienced by the current methods.

This chapter contains a review of current methods based on *seven* desired characteristics, with the first four taken directly from the Russian researchers. We feel that the incorporation of the three additional properties would lead to a more comprehensive and versatile algorithm, thus warranting inclusion in our review. The seven desired characteristics are listed in Table 3.1.

Desired Characteristics
1. Independent of region traversal order
2. Independent of coordinate axes
3. Global displacements per iteration
4. Conformal mapping
5. Intersection prevention
6. Ability to fix points
7. Incorporation of user controls on area vs. shape

Table 3.1: Desired characteristics in a cartogram construction method.

The first two characteristics enable reproducible results. Ideally, a method will produce the same cartogram regardless of the order in which the regions are operated upon or the rotation of the coordinate axes.

In order to be an effective communication tool, the cartogram should enable the recognition of regions through the preservation of the original geographical shapes. Conformal mapping is the preservation of angles locally so that the detailed areas on the cartogram are similar to the original map [4]. We desire to produce transformations that are differentiable, preserving conformality in each small area of the map despite the distortions in shape [6].

The fourth desired property is that the displacement of a map vertex be influenced by all other vertices in the map, not just those adjacent to it. Consider a locomotive attempting to depart with a train of ten cars. The initial pull of the locomotive produces a ripple effect down the train, with the last car feeling the force only after it has rippled through each of the previous nine cars. We desire, instead, a system based on global displacements where *all* regions contribute their influence upon each vertex, at a strength inversely proportional to distance. This generally results in a faster convergence to a solution.

It is also beneficial that the algorithm prevent region intersections, a common occurrence during complex map transformations. Preventing the overlapping of regions is often a significant, time-consuming task since a topological test is required prior to the modification of a vertex. However, intersections must be addressed beyond theory to ensure quality results and, therefore, it is our fifth desired characteristic.

Another desired functionality concerns the user’s ability to restrict vertices from movement. The outer boundary of Texas, for example, could be preserved while only the individual counties within are exaggerated. Incorporating this functionality also enables the user to mandate aesthetic judgments. The user, for example, could make permanent aesthetic modifications in the middle of the cartogram process, forcing all other vertices to adjust accordingly. Allowing immovable vertices permits custom cartograms that conform to the user’s desires.

Finally, the cartogram algorithm should include an appropriate amount of user control in order to satisfy the user’s needs. For example, there is not a unique solution to an equal population cartogram of the United States. On one hand, the cartogram could be perfectly accurate yet totally unrecognizable; on the other hand, the result could be perfectly recognizable yet terribly imprecise. The user needs the freedom to steer between these two extremes; to demand more area convergence over shape, or more exact shapes at the expense of area accuracy.

3.2 *Rubber Map Method* (Tobler, 1973)[1]

In 1973, Waldo Tobler introduced one of the first cartogram methods specifically for use in population districting [23]. He described stretching a "rubber map," upon which every person was represented by a dot, until all the dots were equidistant. On this transformed map all of the "perfect" political districts would appear the same size. Likewise, hexagonal districts could be drawn on the stretched map and be inversely transformed back onto the original map, enabling perfect districting.

Tobler's method initially divides the map into a regular lattice grid, computing a density value for each grid quadrilateral. For each grid vertex, a displacement direction is computed that seeks to minimize the density error of its four adjacent quadrilaterals. This displacement process continues until little or no improvement can be made (i.e. it converges). Since there is a one-to-one mapping between the initial and final grids, Tobler implements a double bivariate interpolation to project map vertices to and from the final grid. The example in Figure 3.1 is a visual example of his method, where the left column is in the original map space and the right column is in the cartogram space.

Tobler's algorithm excels not just in its simplicity of concept but its invariance with respect to the order of region traversal; i.e. the algorithm does not favor any vertices or regions over others. However, the results are very dependent upon the choice of coordinate system. Tobler's calculations require that the vertices be displaced perpendicular to the grid cells, which by default are created parallel to the coordinate axes. In addition, since a vertex is displaced based upon only its four adjacent cells and not the whole map, this method does not converge quickly [10]. The most significant disadvantage of this method is its inaccuracy, with the population cartogram in Figure 3.1 containing 68% error!

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[1] Please note that the names of the methods used here are *descriptive* in nature but may not necessarily be endorsed by each of the authors.

3.3 *DEMP (Radial Expansion) Method* (Selvin et al., 1984)

Seeking a way to visually display and analyze epidemiological data, researchers at Lawrence Berkeley Laboratory created a Density Equalized Map Projection (DEMP) algorithm [20]. The method initially calculates a spatial magnification factor for each region. A region is selected, and a radial transformation is projected outward from its geographic centroid onto all map vertices, such that the transformation multiplies the region's area by its magnification factor while leaving the area of all other regions unchanged. It is interesting to note that the transformation preserves the selected region's *shape* while changing its *area*, and preserves all other region *areas* while changing their *shapes*. Subsequently, another region is selected and a new transformation is created and applied, with this process continuing until the desired equal density map is obtained.

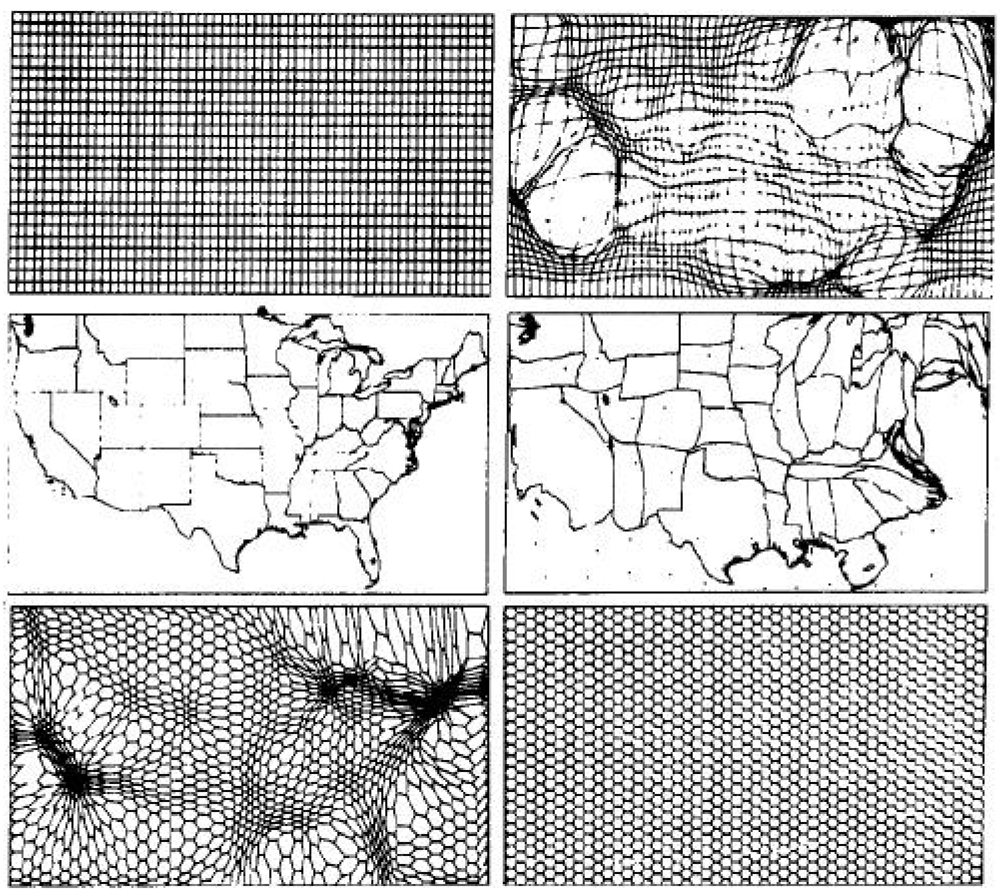
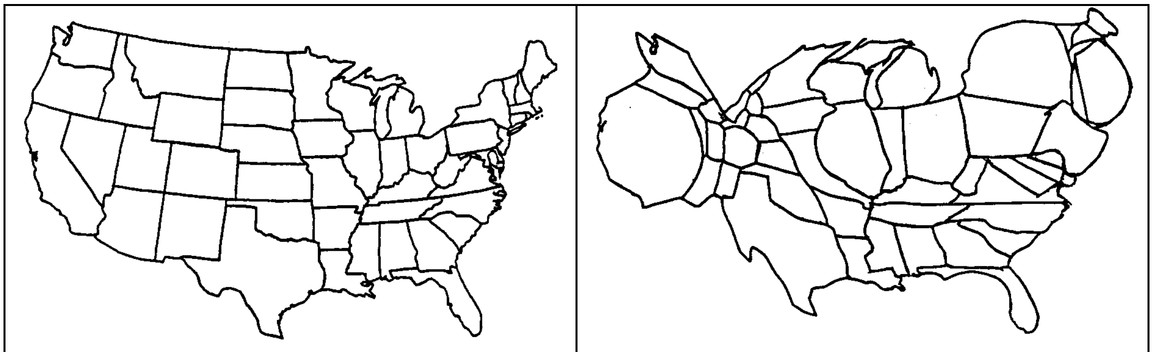


Figure 3.1: U.S. equal population cartogram and equal districting map by Tobler (Reproduced with permission from [23], page 217, Figure 1).

Since the transformations are radial in nature, straight edges of the nonselected regions are warped into curves. The effects can be seen in the radial swelling of the individual states in Figure 3.2.



a) Traditional map. b) Density Equalizing Map Projection (DEMP).

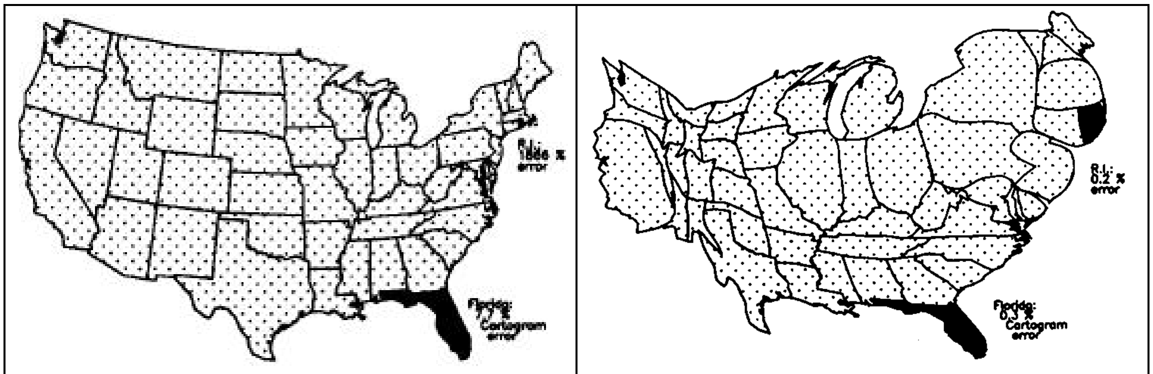
Figure 3.2: U.S. Population density map by Selvin et al. (Reproduced with permission from [20], page 21, figure 3).

By virtue of its radial transformation, the DEMP method will produce the same results regardless of the chosen coordinate system. However, the result depends heavily upon the order of region traversal. In addition, this method employs only local displacement since the translation of the current region's vertices is based solely upon the region itself and not the global map [9].

3.4 *Rubber Sheet Distortion Method* (Dougenik et al., 1985)

Determined to achieve more accurate results than Tobler, three Harvard University graduate students published an algorithm [6] that is based on rubber sheet distortion. Each region exerts a radial force upon *all* the map vertices, with a magnitude proportional to the region area error and inversely proportional to distance. A linearly decreasing force is applied to interior vertices near the region's centroid to prevent instability from enormous forces. Each map vertex is displaced relative to the total force upon it, with the process continuing until the desired equal density map is obtained.

A 1960 population cartogram of the U.S. using this method is shown in Figure 3.3. The significance of this algorithm is that, for the first time, all polygons contribute to the displacement of each vertex. In addition, the results are invariant with respect to both the coordinate system and polygon traversal order [10]. However, the method does not guarantee a non-inverted result since intersections are neither detected nor corrected. It is our view that the results could be improved aesthetically if the user could require better shape preservation or correct undesired artifacts by pinning down modifications.



a) Original map. b) Cartogram.

Figure 3.3: 1960 U.S. population cartogram by Dougenik, et al. (Adapted from and reproduced with permission from [6], pages 78 and 80, figures 3 and 6).

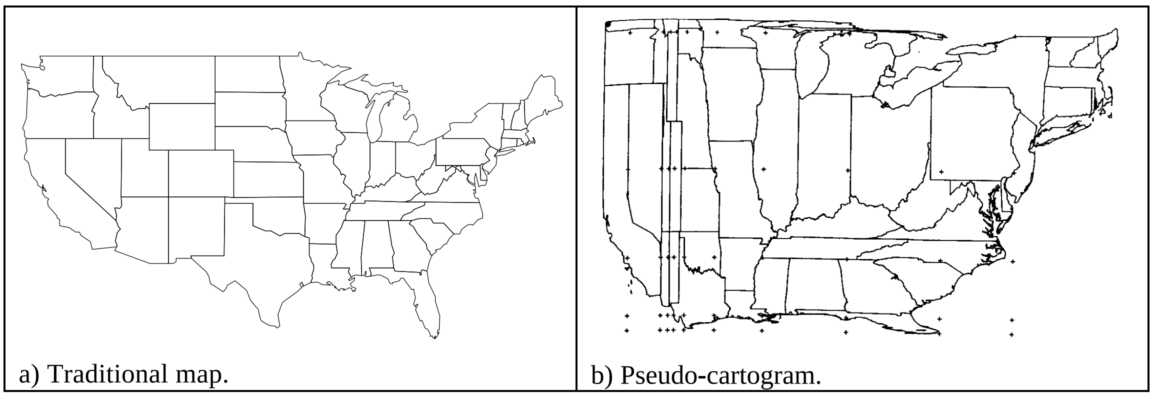
3.5 *Pseudo-Cartogram Method* (Tobler, 1986)

Tobler introduced another method in 1986 that produces pseudo-cartograms [25], or approximations, that provide a convenient starting point for his iterative *Rubber Map* algorithm. This method seeks to reduce the area error by, essentially, adjusting the lines of latitude and longitude on the map. Once a minimum root mean square error solution is obtained, the vertices are translated and scaled relative to the final dimensions of their rectangular section. A sample result of the United States is shown in Figures 3.4.

This method provides a useful tool for effectively minimizing region area error. However, the method heavily depends upon the chosen coordinate axes. Moreover, the method is only designed to “preprocess” a map, optimizing it prior to cartogram construction. Such an approximation can contain extensive error and should not be considered a true cartogram.

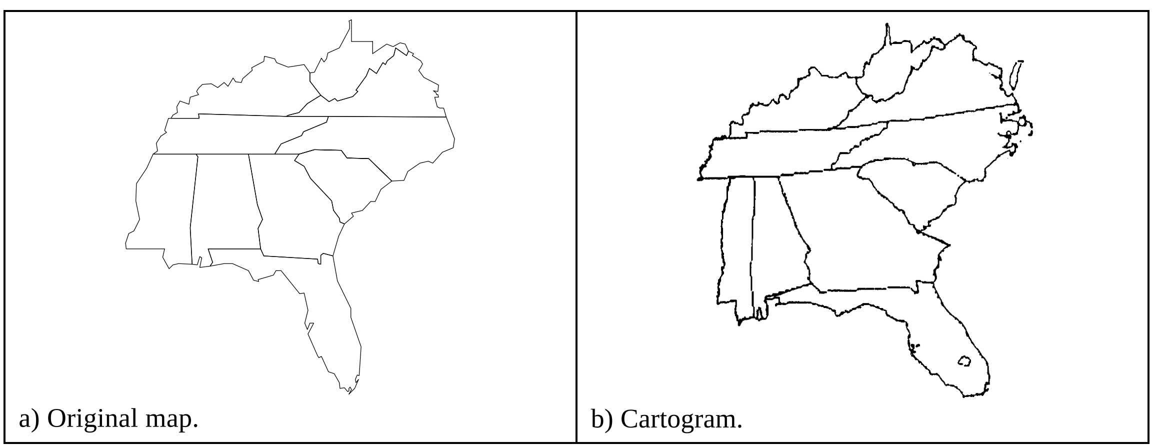
3.6 *Interactive Polygon Zipping Method* (Torguson, 1990)

Desiring to fill the void of user-interactive algorithms, Jeffrey Torguson created an interactive continuous cartogram program, as documented in his 1990 thesis [26]. In his method, the user independently scales and rotates each region along the X and Y axes to achieve desired areas, and positions the resulting regions together as closely as possible. Adjacent region edges are then *"zipped"* together by an edge matching algorithm.



a) Traditional map. b) Pseudo-cartogram.

Figure 3.4: Pseudo-cartogram population density approximation of the U.S. by Tobler (b: Reproduced with permission from [25], page 49, figure 8, © 1986 American Congress on Surveying and Mapping).



a) Original map. b) Cartogram.

Figure 3.5: 1980 population of the southeastern U.S. by Torguson (b: Adapted from and reproduced with permission from [18], page 54, figure 3.13).

With user effort this approach can provide good results, such as the population cartogram of the southeastern U.S. shown in Figure 3.5. However, algorithms should ideally do most of the work and let the user provide final minor adjustments. Torguson's method is excessive in that the user does practically all the work. In addition, Torguson's method does not prevent the user from zipping distant polygons, thus misrepresenting areas and violating the cartogram's integrity. Furthermore, connectivity between regions is not preserved correctly, such as the disjoining of a common vertex between Tennessee and Mississippi as well as between Georgia and South Carolina.

3.7 *Cellular Automata Machine Method* (Dorling, 1990)

A totally different approach to constructing continuous cartograms [4] came in Daniel Dorling's adaptation of the "Game of Life," by John Conway from the University of Cambridge. The map is initially divided into a checkerboard grid, where regions represented by too few grid squares will acquire from those with too many. The algorithm stipulates that corner grid squares, having few fellow peers, are most prone to be traded to another region whereas those with many peers are less likely. The process continues until every region has correctly obtained its desired amount of squares.

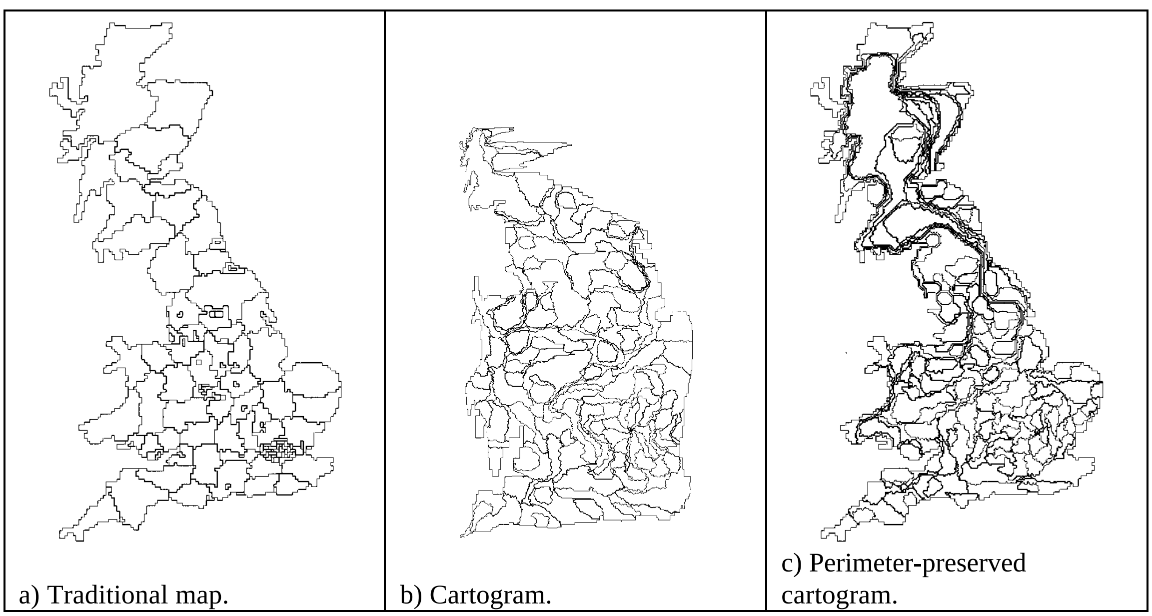
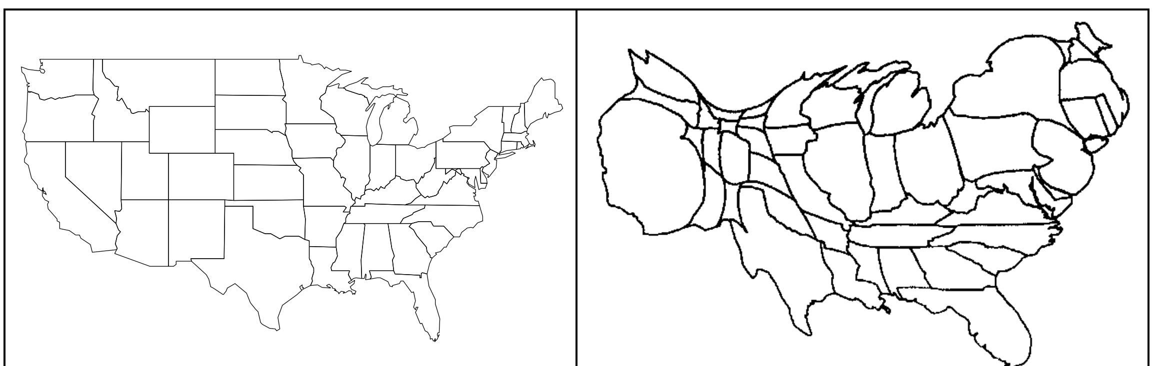


Figure 3.6: Population of British counties and largest cities by Dorling (Reproduced with permission from [4], pages 20-22, figures 14b, 14c, and 14e).

A population density map of Britain using Dorling's method is shown in Figure 3.6. The primary advantage of Dorling's method lies in its ability to fix map vertices. At present, only the "Cellular Automata Machine" algorithm offers this functionality. The method's main shortcoming is its lack of shape preservation. Since cells on the corners of regions are most susceptible to change, regions tend to lose their unique contours and acquire a simpler (and often unrecognizable) shape.

3.8 *Line Integral Method* (Gusein-Zade and Tikunov, 1993)

The two Russian researchers who reviewed current cartogram methods also developed a new method using line integrals [9]. The algorithm initially divides the map into a grid of very small (infinitesimal) cells. A point *P* is chosen within a cell *C* and a radial transformation is applied that makes the density uniform in *C* while leaving the density unchanged at all other points outside *C*. This process is repeated for every point in every cell, changing the shape of the overall map by moving each point in the direction of the vector sum of all the translations applied upon each infinitesimal cell. Since the result will only contain the vertices on the edges of the region boundaries, this transformation is applied in practice as a line integral around each of the region boundaries [1].



a) Traditional Map

b) Cartogram.

Figure 3.7: U.S. population by Gusein-Zade and Tikunov (b: Reproduced with permission from [9], page 172, figure 1, © 1993 American Congress on Surveying and Mapping).

The population cartogram shown in Figure 3.7 was created using this technique. The results are very similar to that of Selvin et al., due to their common radial foundation. Gusein-Zade and Tikunov have successfully produced an algorithm where the results are invariant with respect to both the coordinate system and polygon traversal order. However, we maintain that the results could be improved aesthetically if the user could require better shape preservation or correct undesired artifacts by pinning down modifications.

3.9 Comparison Summary

An overview of the characteristics incorporated into each of the aforementioned cartogram methods is depicted in Table 3.2. While each method achieves various characteristics, none has successfully implemented them all. It is interesting to note that comprehensive intersection prevention was achieved only in Tobler's *Rubber Map Method*, the very first continuous cartogram computer algorithm. Torguson's *Interactive Polygon Zipping Method* stands alone in its incorporation of user control, although to an excessive degree. Dorling's *Cellular* algorithm is the only one that enables immovable vertices, yet is also the only one that does not preserve conformality due to its generalization of region shapes.

Cartogram Characteristics	Tobler (1973) *Rubber Map*	Selvin et al. (1984) *DEMP*	Dougenik et al. (1985) *Rubber Sheet Distortion*	Tobler (1986) *Pseudo-Cartogram*	Torguson (1990) *Inter. Polygon Zipping*	Dorling (1990) *Cellular Automata Machine*	Gusein-Zade, Tikunov (1993) *Line Integral*
1. Independent of region traversal order	**YES**	No	**YES**	**YES**[1]	**YES**[1]	**YES**[1]	**YES**
2. Independent of coordinate axes	No	**YES**	**YES**	No	No	**YES**	**YES**
3. Global displacements per iteration	No	No[3]	**YES**	**YES**	**YES**[1]	No	**YES**
4. Conformal mapping	**YES**	**YES**	**YES**	**YES**	**YES**	No	**YES**
5. Intersection prevention	**YES**[2]	No	No[4]	**YES**[1]	**YES**[1]	**YES**[1]	No[6]
6. Ability to fix points	No	No	No	No	No	**YES**	No
7. Incorporation of user controls on area vs. shape	No	No	No	No	**YES**[5]	No	No

[1] Not applicable and, therefore, satisfies this characteristic.

[2] A time-intensive topological test prevents the vertices from crossing [23].

[3] Each transformation adjusts the area of only one region without concern for the other regions [10].

[4] Although placing extra vertices at regular intervals and splitting regions into convex sections effectively prevents self-intersection [6].

[5] Although the user has *too much control* and practically does *all* the work.

[6] Self-intersection can be avoided by adding more vertices or decreasing the time step, thus increasing computation time [15].

Table 3.2: Summary of characteristics of the various cartogram methods.